Sufficient Conditions for Stability of the Bernard Problem in Arbitrary Geometries

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Recently Catton and Edwards (*1*) and Davis (*2*) have used linearized stability analyses to find necessary conditions for stability of Newtonian fluids heated from below and contained in right circular cylinders or three-dimensional boxes, respectively. We wish to point out that the energy method of stability analysis (*3*, *4*) can be used to prove that the sufficient conditions for stability coincide with the necessary conditions for stability for the geometries and boundary conditions considered in references *1* and *2*.

In fact, the energy method can be used to show that the necessary and sufficient conditions for stability coincide for arbitrary geometries if the following conditions are met (*3*):

1. The Boussinesq approximation is valid.
2. A stationary-state solution exists with a zero velocity and a temperature gradient that is linear, parallel to, and in the same direction as gravity.
3. The velocity perturbations satisfy either $V_j = 0$, or $V_i \cdot N_i = 0$, $V_j \, \partial V_j / \partial X_i \, N_i = 0$ at all finite boundaries.
4. The temperature perturbations satisfy the general equation $\partial T' / \partial X_i \, N_i + N_{Nu} T' = 0$ at all finite boundaries.
5. In any infinite directions the velocity and temperature perturbations are either periodic or decay asymptotically to zero. When these criteria are satisfied, the results obtained from a linear stability analysis with exchange of stabilities are necessary, and sufficient conditions for stability, and subcritical instabilities cannot occur.

NOTATION

N_{Nu} = Nusselt number
N_i = dimensionless outward directed normal
T' = dimensionless perturbation in temperature
V_j' = dimensionless perturbation in velocity
X_i = dimensionless general coordinate direction

LITERATURE CITED

1. Catton, I., and D. K. Edwards, *AIChE J.*, **16,** 594 (1970).
2. Davis, S. H., *J. Fluid Mech.*, **30,** 465 (1967).
3. Joseph, D. D., *Arch. Ratl. Mech. Anal.*, **20,** 59 (1965).
4. Shir, C. C., and D. D. Joseph, *Arch. Ratl. Mech. Anal.*, **30,** 38 (1968).

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comprehension by novices difficult. The names of authors of literature references are often misspelled and in some cases are indexed twice or not at all.

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Gas-Liquid Reactions, P. V. Danckwerts, McGraw-Hill Book Co., New York (1970). 276 pages. $11.50.

Gas-liquid absorption in both reacting and nonreacting systems has been the subject of numerous experimental and theoretical studies in the past twenty years. The author has been noteworthy among those who have applied our increased understanding of kinetics and mass transfer to the development of coherent models for the design of gas-liquid contacting apparatus.

This book summarizes the results of such advances both by his own group and by many others. As a clear presentation of the subject from basic principles to design of process units, the book represents a significant contribution to the chemical engineering literature. It is suitable as a text for a graduate course in mass transfer operations and will also be valuable as a reference for the practicing engineer, because of the clear exposition of underlying theory and because of the large number of practical examples.

Although not so designated by the author, the book falls into three sections. The first four chapters discuss *the elements of diffusion* and absorption into quiescent liquids with simultaneous reaction obeying various kinetic laws. The second section discusses absorption into agitated liquids, an area in which the author has made significant contributions. Finally, the last four chapters discuss absorption with reaction in process equipment.

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The introduction to the book offers a list, with extensive references, of examples of industrially important processes in which gas absorption is accompanied by chemical reaction. Chapter one presents basic principles of diffusion and solubility in liquids. Of particular value here is a discussion of diffusion of electrolytes and of the effect of mixed electrolytes on diffusivity of the absorbing components. Chapter two gives some of the basic principles of reaction kinetics. In Chapter three, "Absorption into Quiescent Liquids," the author presents analytical solutions for absorption with reactions of first, second, and general order, both for irreversible and reversible reactions. Plots of enhancement factors for the various reaction cases, that is, the increase in absorption coefficient due to a chemical reaction over the nonreaction case, are given for a wide range of parameters. Many examples with completely calculated numerical results show how to use the results as well as the particular limitations of each of the solutions.

Chapter four discusses use of these solutions in analysis of results of experiments with apparatus, such as the laminar jet, wetted sphere, and wetted-wall column, and is particularly valuable for the discussion of the errors that may arise and caveats to be observed in interpreting data from each of these types of apparatus.

The next three chapters discuss similar topics as applied to absorption and reaction in agitated liquids. Various film, surface renewal models, and the variable age model associated with the author's name are discussed in both nonreacting and reacting systems. Examples and complete solutions for different kinetics are given. Comparison of results using film, Higbie, and Danckwerts' models are given for first-order reactions as well as for instantaneous irreversible reactions and for irreversible second-order reactions. Temperature effects and absorption with ionized reaction products also are discussed. As a compilation with numerous examples and references to the different types of models of absorption with reaction, this chapter alone is worth the price of the book.

The final three chapters discuss application to process equipment, including absorption in packed and plate columns, and the characteristics of various types of absorption equipment. Chapter ten, entitled "Some Examples of Reactions," gives complete analysis with numerical results for a number of practical absorbers and will be of particular interest to the practicing engineer. Some of the examples discussed are: absorption of carbon dioxide in various alkali solutions, absorption of chlorine in aqueous solutions, and oxygen reacting in reducing aqueous solutions. The book concludes with a chapter on desorption of gases.

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LETTERS TO THE EDITOR

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to believe that the dependence of K' on β for other fluids (e.g., dilute polymer solutions and Newtonian fluids) would also yield a similar trend to that shown in Figure 2.

NOTATION

D = tube diameter, in.
g_c = conversion factor, 32 $\text{lb.}_m\text{-ft./lb.}_f\text{-sq.sec.}$
K' = dimensionless coefficient
K_p = fluid consistency in the power law, $\text{lb.}_m\text{/in.-sec.}^{2-n}$
L/D = capillary length-to-diameter ratio
n = flow index in the power law, dimensionless
N_{Re} = Reynolds number, dimensionless
ΔP_{ent} = entrance pressure drop, $\text{lb.}_f\text{/sq.in.}$
V = average velocity of fluid, in./sec.
ρ = fluid density, $\text{lb.}_m\text{/cu.in.}$
β = ratio of capillary cross-section area to reservoir cross-section area

LITERATURE CITED

1. Han, C. D., unpublished work.
2. Han, C. D., M. Charles, and W. Philippoff, *Trans. Soc. Rheol.*, **13**, 455 (1969).
3. *Ibid.*, **14**, 393 (1970).
4. Sylvester, N. D., and S. L. Rosen, *AIChE. J.*, **16**, 964 (1970).
5. Weissberg, H. L., *Phys. Fluids*, **5**, 1033 (1962).

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TO THE EDITOR

In the interest of maintaining the quality of the *Journal*, I feel it necessary to call attention to a minor error in the recent article by Truesdell and Adler, "Numerical treatment of fully developed laminar flow in helically coiled tubes" (*1*). These authors repeatedly refer to "the Deans number" when consistency would require "the Dean number" after the original investigator (*2*). The terminology "Reynolds number," "Prandtl number," and "Schmidt number" has been used, at least in this country, since 1941 or earlier (*3*).

Curiously, Koutsky and Adler's "listing of important references through 1964" (*4*), as it is described in the subject article, makes only one peripheral reference to this dimensionless group. There it is called "a . . . Dean's number" and not defined. White, in his early paper (*5*), uses the phraseology "Reynolds number" but "Dean's criterion." Dean himself did not, of course, venture to give this dimensionless group a name.

Since it is too much to ask you or your associate editors to catch pecadillos of this kind, I appeal to your authors and reviewers for more care.

LITERATURE CITED

1. Truesdell, L. C., Jr. and R. J. Adler, *AIChE J.* **16**, 1010 (1970).
2. Dean, W. R., *Phil. Mag.* **4**, 208 (1927); **5**, 673 (1928).
3. Perry, J. H., Ed., "Chemical Engineers' Handbook," McGraw-Hill, New York (1941); also *American Standard* ASA Y10.12-1955.
4. Koutsky, J. A., and R. J. Adler, *Can. J. Chem. Eng.* **42**, 239 (1964).
5. White, C. M., *Proc. Roy. Soc.* (*London*) **A 123**, 645 (1929).

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